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A simulated annealing algorithm for discrete minimal weight design of shallow space trusses with stability constraints

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Abstract

A new simulated annealing algorithm is proposed for discrete minimal weight design of shallow space trusses. The process allows us to consider the global and local stability problems as well. Because of the highly nonlinear phenomena, it is essential to use appropriate and safe models that detect these effects as accurately as possible. In this paper, a non-linear path-following method is applied for the analysis of non-linear stability problems. The design variables are the cross-sectional areas selected from a given set of cross-sections. In the simulated annealing procedure, a truss structure is characterized by its cross-sectional areas, its weight, and its maximal constraint violation free load intensity factor. A random neighbor is defined to be any truss that can be created by replacing the cross-sectional area of one element group in the current truss by its immediate predecessor or successor area.

To demonstrate the efficiency of the simulated annealing method proposed, two numerical examples, a 24-member, and a 30-member dome structure are presented where structural instability constraints and member buckling are considered as well. Computational results indicate that the simulated annealing procedure is a viable approach for a very difficult structure optimization problem.

1 Introduction

There are several exact and heuristic methods proposed solving optimization problems with continuous and discrete design variables, see e.g. Refs. [1,9,10]. Two optimization techniques were studied to solve mixed reliability-based optimization problem by Kleiber et al. [6], and Stocki et al. [11]. An equivalent-continuous optimization problem and alternatively the controlled enumeration method have been presented. The main advantage of the second method is that free from the convergence problems observed with the first algorithm for large number of design variables. The main disadvantage of the pure enumeration method [8] that without any pruning rule it is applicable only for trivial cases. To avoid this difficulties a new implicit enumeration method was presented see Refs. [3,4,5], where the discrete optimal design method was formulated as an exterior-point tree-search problem that maintains a lower bound to allow us to discard the solutions if its weight is greater than or equal to the current lower bound.

In this paper new simulated annealing algorithm is presented for shallow space trusses with stability constraints. The simulated annealing algorithm has proven to be a good technique [7] for solving combinatorial optimization problems in particular for large flexible space structures. However it seems sometimes less useful than some conventional algorithms. Consequently, simulated annealing has not been widely accepted in engineering optimization. In order to accelerate the overall convergence, it is proposed to use the best solution for a starting point every time that the temperature is reduced. The results showed that simulated annealing algorithm provides a computationally efficient tool find near optimal solutions otherwise computationally prohibited problems.

2 Discrete optimization problem

A new simulated annealing algorithm is discussed for discrete minimal weight design of shallow space trusses with stability constraints. A higher-order path-following method [2] is involved to detect the structural instability points in terms of an arc length parameter.

2.1 Problem formulation

The geometrically nonlinear space truss structure is formulated as a large displacement model, where the total potential energy function can be described in the following way:

$$V(u_i, a_q, \lambda) = U(u_i(a_i)) - \lambda p_i u_i \tag{1}$$

$$i = 1,2,\ldots,n \qquad q = 1,2,\ldots,e \tag{2}$$

where λ is the load intensity parameter, p_i is the applied external load vector, u_i is the nodal displacement vector, a_q is the vector of the member cross-sectional areas, and n is the number of nodes, e is the number of elements. The $U(u_i(a_q))$ function is the non-linear strain energy function supposed only linear elastic material.

The discrete optimization problem is discussed in terms of the nodal displacements and the cross section area of the truss members. The design variables a_q are selected from a discrete set of the predetermined $a_q \in A = \{A_1, A_2, ..., A_Q\}$ cross-sectional areas, such that minimize the total weight of the structure:

$$w(a_q) \to \min! \tag{3}$$

subject to the

$$V_{,i} = 0 \tag{4}$$

$$\eta_i(V_{,ij}) \geq 0 \tag{5}$$

$$\underline{s} \leq s_q \leq \bar{s}_q \tag{6}$$

$$i = 1,2,...,n \qquad q = 1,2,...,e \qquad j = 1,2,...,e \tag{7}$$

where $V_{,i} = 0$ is the equilibrium criterion, η_i is the vector of eigenvalues of the $V_{,ij}$ Hessian, and $\underline{s}$, $\bar{s}_q$ are the lower and upper bounds of the stress constraints.

2.2 Stability constraints

The proposed instability investigation [2] is based on the perturbation technique of the stability theory and on the non-linear modification of the classical linear homotopy method. This higher order predictor-corrector method provides an accurate computation of the singular points. It is capable to compute not only points but also segments of the equilibrium path. The curve segment approximation is the base of investigation of the singular points. Since we are concerned with finding feasible designs we must define a certain appropriate measure of performance. In the proposed path-following approach the measure of design unfeasibility is defined as follows:

$$\lambda(t) \to \max! \tag{8}$$

$$0 \leq \lambda(t) \leq 1 \tag{9}$$

$$\eta_i(t) > 0 \tag{10}$$

$$\underline{s} \leq s_q(t) \leq \bar{s}_q \tag{11}$$

$$i = 1,2,...,n \qquad q = 1,2,...,e \tag{12}$$

where t is the arch-length parameter of the equilibrium path.

3 Simulated annealing algorithm

Simulated annealing is a computational process, which attempts to solve hard combinatorial optimization problems through controlled randomization. Simulated annealing emulates the physical process of annealing which attempts to force a system to its lowest energy state trough controlled cooling.

In general, the annealing process involves the following steps:

1. The temperature of the system is raised to a sufficient level.
2. The temperature of the system is maintained at the level for a prescribed amount of time.
3. The system is allowed to cool under controlled conditions until the desired energy state is attained.

The initial temperature the time system remains at this temperature and the rate at which the system is cooled are referred to as the annealing schedule. If the system is allowed to cool too fast it may freeze at an undesirable high-energy state. The state of system corresponds to the actual value of the objective function. The freezing of a system at an undesirable energy state corresponds to an optimization problem, which is unable to leave a local optimum point. In simulated annealing the process starts at a given feasible or unfeasible solution. A series of moves (changes of values of decision variables) are made according to the given annealing schedule until either the optimal solution is attained or the problem becomes frozen at a local optimum which it cannot leave. To avoid freezing at a local optimum the algorithm walks very slowly (with very small objective value changes) trough the solution space. The controlled improvement of the objective value is accomplished by accepting non-improving moves with a certain probability, which decreases as the algorithm progresses.

The general procedure for implementing a simulated annealing algorithm may be described as follows:

```
0: MaxStep = 1000:  MaximalNonImprovingPasses = 10: Seed = StartingSeed
  Call ProblemDefinition:  Step = 0: Node = 0: Temperature = 1: CoolingRatio = 0.95
  Call RandomInitialStructure: BestWeight = Null: ReDim BestArea(Groups)
  ReDim Areas(Groups, Node), Weight(Node), LoadFactor(Node), Cooling(Node)
  ReDim ParentNode(Node), LogicalStep(Node), LogicalNode(Step)
  Call FollowMe: Weight(Node) = WeightOfStructure: LoadFactor(Node)=LoadIntensityFactor
  For i = 1 To Groups: Areas(i, Node) = Area(i): Next i: ParentNode(Node) = Null
  LogicalStep(Node) = Step: LogicalNode(Step) = Node: Cooling(Node) = Temperature
  If FeasibleStructure Then BestWeightUpdate
1: For Step = 1 To MaxStep
  Call TemperatureUpdate: If Temperature < 0.001 Then  Exit Sub
  ReDim Preserve LogicalNode(Step)
  For n = 1 To MaximalNonImprovingPasses
  If RandomNeighbourStructure Then
        Node = Node + 1
        ReDim Preserve Areas(Groups, Node), Weight(Node), LoadFactor(Node), Cooling(Node)
        ReDim Preserve ParentNode(Node), LogicalStep(Node)
        LogicalStep(Node) = Step: ParentNode(Node) = LogicalNode(Step - 1)
        LogicalNode(Step) = Node: Cooling(Node)=Temperature
        Call
        FollowMe:Weight(Node)=WeightOfStructure:LoadFactor(Node)=LoadIntensityFactor
        For i = 1 To Groups: Areas(i, Node) = Area(i): Next i
        If FeasibleStructure Then BestWeightUpdate
        If LoadFactor(Node) > LoadFactor(LogicalNode(Step - 1)) Then GoTo 2
        Probability=Exp(-Abs(LoadFactor(Node)-LoadFactor(LogicalNode(Step-1)))/Temperature)
        If UniformRandomNumber < Probability Then GoTo 2
Else
  Exit Sub
End If
Next n
2: Next Step
```

4 Numerical examples

The results obtained using simulated annealing method has been illustrated the effectiveness of the proposed method for medium-size structural design problems. Two of the permanently used test examples are considered. The geometry of the 24-member and the 30-member truss dome are shown in Fig.1 and Fig.2. The elasticity modulus is $E = 7x10^{10}\ N/m^2$. The stress constraints are for tension and compression $25x10N/m^2$. The density is $27500N/m^3$. According to the requirement of the symmetrical structure, both cases the members were partitioned into linking groups.

4.1 The 24-member dome structure

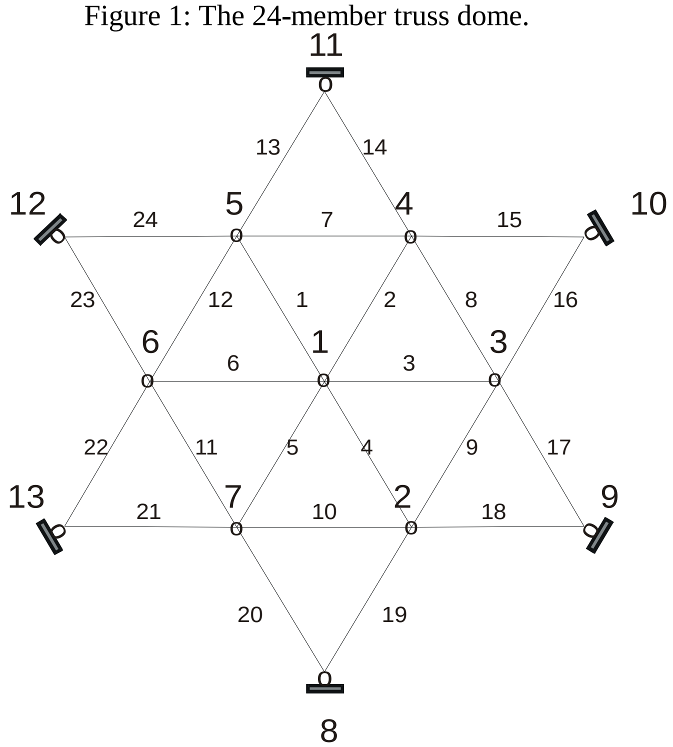
The cross-sectional areas of the truss-members with circular sections are selected from an available catalogue: $A_i \in \{12{,}57;15{,}90;19{,}63;23{,}76;28{,}27;33{,}18;38{,}48;44{,}18;50{,}27\}x10^{-4}m^2$.

Table 1: Geometry of the .24-member dome structure

nodal points	**x** [m]	**y** [m]	**z** [m]
1	0	0	0
3	25	0	2
4	12.5	21.65	2
10	43.3	25	8.216
11	0	50	8.216

The applied loads of the 24-member dome structure are $P_1 = 6kN$ at the nodal point 1, and $P_{2-7} = 12kN$ at the nodal points 2-7, that causes a bifurcation instability phenomena. The results of the optimization process are shown in Table 3 and Table 4.

Figure 1: The 24-member truss dome.



4.2 The 30-member dome structure

The cross-sectional areas of the truss-members with circular sections are selected from the catalogue: $A_i \in \{12,57;15,90;19,63;23,76;28,27;33,18;38,48;44,18;50,27;56,75;63,62;70,88;78,54\} x10^{-4} m^2$ which is larger than the catalogue used for the 24-member truss dome.

Table 2: Geometry of the .30-member dome structure

nodal points	**x** [m]	**y** [m]	**z** [m]
1	0	0	65.041
3	300.000	0	48.923
4	150.000	259.808	48.923
11	600.000	0	0
12	450.000	259.808	16.404
13	300.000	519.615	0
14	0	519.615	16.404

The applied load of the 30-member dome structure is $P_1 = 15kN$ which acts at the nodal point 1. Therefore, the structure exhibits a snap-through, or limit point instability loss. The results of the optimal design of 30-member dome structure are demonstrated in Table 5 and Table6.

Figure 2: The 30-member truss dome.

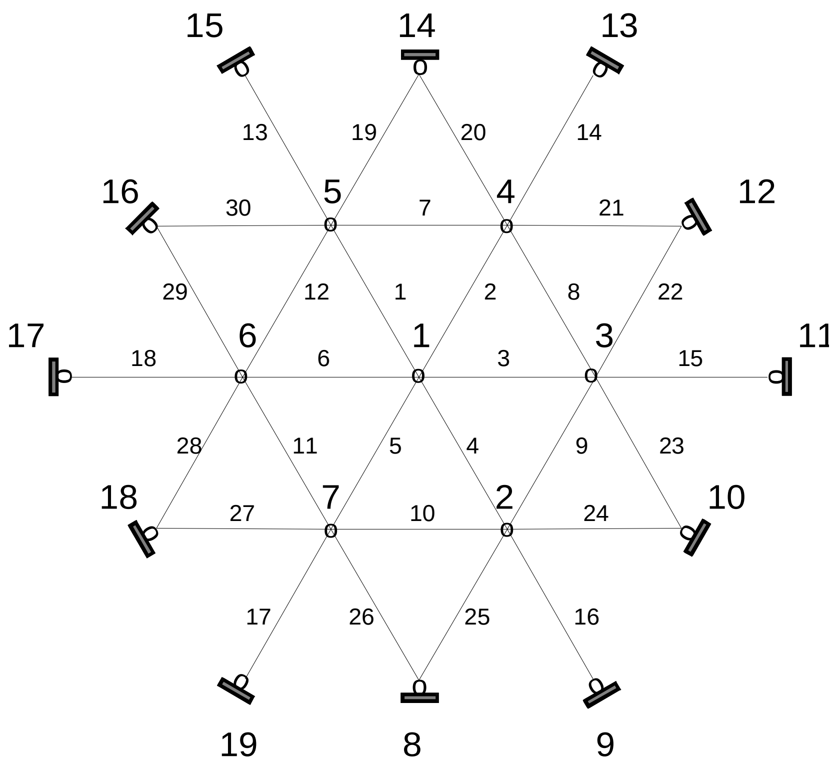


Table 3: The results of the 24-member dome structure

		Cross Sections		Temp	Load	Weight	Best Weight
17	17	{12.57,19.63,12.57}	{1,3,1}	0.418	1.000	264.068F	264.068
16	16	{12.57,15.90,12.57}	{1,2,1}	0.440	0.971	248.682	269.671
15	15	{15.90,15.90,12.57}	{2,2,1}	0.463	0.999	262.462	269.671
14	14	{15.90,12.57,12.57}	{2,1,1}	0.488	0.938	248.726	269.671
13	13	{12.57,12.57,12.57}	{1,1,1}	0.513	0.910	234.946	269.671
12	12	{12.57,12.57,15.90}	{1,1,2}	0.540	1.000	269.671F	269.671
11	11	{15.90,12.57,15.90}	{2,1,2}	0.569	1.000	283.451F	283.451
10	10	{19.63,12.57,15.90}	{3,1,2}	0.599	1.000	298.886F	298.886
9	9	{19.63,12.57,12.57}	{3,1,1}	0.630	0.960	264.161	320.232
8	8	{23.76,12.57,12.57}	{4,1,1}	0.663	0.978	281.251	320.232
7	7	{28.27,12.57,12.57}	{5,1,1}	0.698	0.992	299.914	320.232
6	6	{33.18,12.57,12.57}	{6,1,1}	0.735	1.000	320.232F	320.232
5	5	{33.18,15.90,12.57}	{6,2,1}	0.774	1.000	333.968F	333.968
4	4	{33.18,19.63,12.57}	{6,3,1}	0.815	1.000	349.354F	349.354
3	3	{38.48,19.63,12.57}	{7,3,1}	0.857	1.000	371.287F	371.287
2	2	{38.48,19.63,15.90}	{7,3,2}	0.903	1.000	406.012F	406.012
1	1	{38.48,19.63,19.63}	{7,3,3}	0.950	1.000	444.908F	444.908
0	0	{38.48,19.63,23.76}	{7,3,4}	1.000	1.000	487.975F	487.975

Table 4: The results-tree of the 24-member dome structure

0 (0) {38.48,19.63,23.76} {487.975,1.000,1.000}
1 (1) {38.48,19.63,19.63} {444.908,1.000,0.950}
2 (2) {38.48,19.63,15.90} {406.012,1.000,0.903}
3 (3) {38.48,19.63,12.57} {371.287,1.000,0.857}
4 (4) {33.18,19.63,12.57} {349.354,1.000,0.815}
5 (5) {33.18,15.90,12.57} {333.968,1.000,0.774}
6 (6) {33.18,12.57,12.57} {320.232,1.000,0.735}
7 (7) {28.27,12.57,12.57} {299.914,0.992,0.698}
8 (8) {23.76,12.57,12.57} {281.251,0.978,0.663}
9 (9) {19.63,12.57,12.57} {264.161,0.960,0.630}
10 (10) {19.63,12.57,15.90} {298.886,1.000,0.599}
11 (11) {15.90,12.57,15.90} {283.451,1.000,0.569}
12 (12) {12.57,12.57,15.90} {269.671,1.000,0.540}
13 (13) {12.57,12.57,12.57} {234.946,0.910,0.513}
14 (14) {15.90,12.57,12.57} {248.726,0.938,0.488}
15 (15) {15.90,15.90,12.57} {262.462,0.999,0.463}
16 (16) {12.57,15.90,12.57} {248.682,0.971,0.440}
17 (17) {12.57,19.63,12.57} {264.068,1.000,0.418}

Table 5: The results of the 30-member dome structure

		Cross Sections	Temp	Load	Weight	Best Weight
53	76	{50.27,19.63,15.90,19.63} {9,3,2,3}	0.066	0.946	6215.838	6261.706
53	75	{50.27,19.63,12.57,19.63} {9,3,1,3}	0.066	0.921	6048.825	6261.706
53	74	{50.27,15.90,12.57,19.63} {9,2,1,3}	0.066	0.737	5864.190	6261.706
53	73	{50.27,15.90,15.90,19.63} {9,2,2,3}	0.066	0.767	6031.203	6261.706
53	72	{50.27,15.90,15.90,15.90} {9,2,2,2}	0.066	0.768	5659.770	6261.706
53	71	{50.27,15.90,19.63,15.90} {9,2,3,2}	0.066	0.774	5846.844	6261.706
52	70	{50.27,19.63,19.63,15.90} {9,3,3,2}	0.069	0.954	6031.479	6261.706
52	69	{44.18,19.63,19.63,15.90} {8,3,3,2}	0.069	0.704	5729.589	6261.706
52	68	{44.18,19.63,19.63,19.63} {8,3,3,3}	0.069	0.704	6101.022	6261.706
52	67	{44.18,19.63,15.90,19.63} {8,3,2,3}	0.069	0.705	5913.948	6261.706
52	66	{44.18,19.63,15.90,15.90} {8,3,2,2}	0.069	0.705	5542.515	6261.706
51	65	{50.27,19.63,15.90,15.90} {9,3,2,2}	0.073	0.948	5844.405	6261.706
50	64	{50.27,19.63,12.57,15.90} {9,3,1,2}	0.077	0.776	5677.393	6261.706
50	63	{50.27,19.63,12.57,12.57} {9,3,1,1}	0.077	0.742	5345.792	6261.706
49	62	{50.27,19.63,15.90,12.57} {9,3,2,1}	0.081	0.950	5512.804	6261.706
48	61	{56.75,19.63,15.90,12.57} {10, 3, 2, 1}	0.085	0.957	5834.026	6261.706
48	60	{56.75,19.63,12.57,12.57} {10, 3, 1, 1}	0.085	0.748	5667.014	6261.706
48	59	{56.75,23.76,12.57,12.57} {10, 4, 1, 1}	0.085	0.797	5871.449	6261.706
48	58	{56.75,28.27,12.57,12.57} {10, 5, 1, 1}	0.085	0.839	6094.694	6261.706
47	57	{56.75,28.27,15.90,12.57} {10, 5, 2, 1}	0.090	1.000	6261.706 F	6261.706
46	56	{56.75,33.18,15.90,12.57} {10, 6, 2, 1}	0.094	1.000	6504.751 F	6504.751
45	55	{56.75,33.18,15.90,15.90} {10, 6, 2, 2}	0.099	1.000	6836.352 F	6836.352
45	54	{50.27,33.18,15.90,15.90} {9,6,2,2}	0.099	0.806	6515.130	7040.773
45	53	{50.27,33.18,15.90,19.63} {9,6,2,3}	0.099	0.806	6886.563	7040.773
45	52	{50.27,33.18,12.57,19.63} {9,6,1,3}	0.099	0.807	6719.550	7040.773
44	51	{56.75,33.18,12.57,19.63} {10, 6, 1, 3}	0.105	1.000	7040.773 F	7040.773
43	50	{56.75,33.18,12.57,23.76} {10, 6, 1, 4}	0.110	1.000	7452.038 F	7452.038
42	49	{50.27,33.18,12.57,23.76} {9,6,1,4}	0.116	0.806	7130.815	
41	48	{50.27,28.27,12.57,23.76} {9,5,1,4}	0.122	0.806	6887.771	
40	47	{50.27,28.27,12.57,19.63} {9,5,1,3}	0.129	0.806	6476.505	
39	46	{50.27,28.27,12.57,15.90} {9,5,1,2}	0.135	0.806	6105.072	
38	45	{44.18,28.27,12.57,15.90} {8,5,1,2}	0.142	0.704	5803.183	
37	44	{44.18,28.27,12.57,12.57} {8,5,1,1}	0.150	0.705	5471.582	
36	43	{50.27,28.27,12.57,12.57} {9,5,1,1}	0.158	0.806	5773.471	
35	42	{50.27,23.76,12.57,12.57} {9,4,1,1}	0.166	0.789	5550.227	
34	41	{44.18,23.76,12.57,12.57} {8,4,1,1}	0.175	0.704	5248.337	
33	40	{44.18,23.76,15.90,12.57} {8,4,2,1}	0.184	0.703	5415.349	
32	39	{44.18,19.63,15.90,12.57} {8,3,2,1}	0.194	0.704	5210.914	
31	38	{44.18,19.63,12.57,12.57} {8,3,1,1}	0.204	0.705	5043.902	

Table 6: The results of the 30-member dome structure

		Cross Sections	Temp	Load	Weight	Best Weight
30	37	{44.18,19.63,12.57,15.90} {8,3,1,2}	0.215	0.706	5375.503	
30	36	{38.48,19.63,12.57,15.90} {7,3,1,2}	0.215	0.610	5092.946	
30	35	{38.48,15.90,12.57,15.90} {7,2,1,2}	0.215	0.613	4908.311	
30	34	{33.18,15.90,12.57,15.90} {6,2,1,2}	0.215	0.524	4645.583	
30	33	{33.18,15.90,12.57,19.63} {6,2,1,3}	0.215	0.524	5017.016	
30	32	{38.48,15.90,12.57,19.63} {7,2,1,3}	0.215	0.614	5279.744	
29	31	{44.18,15.90,12.57,19.63} {8,2,1,3}	0.226	0.716	5562.301	
28	30	{44.18,15.90,12.57,15.90} {8,2,1,2}	0.238	0.705	5190.868	
27	29	{44.18,15.90,12.57,12.57} {8,2,1,1}	0.250	0.679	4859.267	
26	28	{44.18,12.57,12.57,12.57} {8,1,1,1}	0.264	0.609	4694.432	
25	27	{44.18,12.57,12.57,15.90} {8,1,1,2}	0.277	0.607	5026.033	
24	26	{38.48,12.57,12.57,15.90} {7,1,1,2}	0.292	0.600	4743.476	
23	25	{38.48,12.57,12.57,12.57} {7,1,1,1}	0.307	0.602	4411.875	
22	24	{38.48,12.57,15.90,12.57} {7,1,2,1}	0.324	0.607	4578.887	
21	23	{33.18,12.57,15.90,12.57} {6,1,2,1}	0.341	0.529	4316.159	
20	22	{33.18,12.57,12.57,12.57} {6,1,1,1}	0.358	0.530	4149.146	
19	21	{33.18,12.57,12.57,15.90} {6,1,1,2}	0.377	0.530	4480.748	
18	20	{33.18,12.57,12.57,19.63} {6,1,1,3}	0.397	0.531	4852.181	
17	19	{33.18,12.57,15.90,19.63} {6,1,2,3}	0.418	0.530	5019.193	
16	18	{33.18,12.57,15.90,23.76} {6,1,2,4}	0.440	0.530	5430.458	
15	17	{38.48,12.57,15.90,23.76} {7,1,2,4}	0.463	0.604	5693.187	
15	16	{38.48,12.57,15.90,28.27} {7,1,2,5}	0.463	0.604	6142.292	
15	15	{44.18,12.57,15.90,28.27} {8,1,2,5}	0.463	0.611	6424.849	
14	14	{44.18,15.90,15.90,28.27} {8,2,2,5}	0.488	0.716	6589.684	
13	13	{44.18,15.90,19.63,28.27} {8,2,3,5}	0.513	0.715	6776.757	
12	12	{50.27,15.90,19.63,28.27} {9,2,3,5}	0.540	0.771	7078.647	
11	11	{50.27,15.90,23.76,28.27} {9,2,4,5}	0.569	0.796	7285.783	
10	10	{50.27,15.90,23.76,23.76} {9,2,4,4}	0.599	0.796	6836.677	
9	9	{50.27,15.90,23.76,19.63} {9,2,4,3}	0.630	0.796	6425.412	
8	8	{50.27,19.63,23.76,19.63} {9,3,4,3}	0.663	0.809	6610.047	
7	7	{56.75,19.63,23.76,19.63} {10, 3, 4, 3}	0.698	0.969	6931.270	
6	6	{56.75,19.63,23.76,15.90} {10, 3, 4, 2}	0.735	0.970	6559.837	
5	5	{56.75,19.63,23.76,12.57} {10, 3, 4, 1}	0.774	0.971	6228.236	
4	4	{56.75,19.63,19.63,12.57} {10, 3, 3, 1}	0.815	0.966	6021.100	
3	3	{50.27,19.63,19.63,12.57} {9,3,3,1}	0.857	0.809	5699.878	
2	2	{50.27,19.63,23.76,12.57} {9,3,4,1}	0.903	0.808	5907.013	
1	1	{50.27,19.63,23.76,15.90} {9,3,4,2}	0.950	0.809	6238.614	
0	0	{50.27,19.63,28.27,15.90} {9,3,5,2}	1.000	0.808	6464.808	

5 Conclusion

In this paper, a simulated annealing algorithm is proposed involving path-following method [2] that provides an accurate computation of each type of the critical points. The results showed that simulated annealing algorithm provides a computationally efficient heuristic algorithm find near optimal solutions. In comparison with the results of the implicit enumeration method we obtained a near optimal solution (BestWeight=6261.706; OptimalWeight=6038.461) for the 30-member truss dome after evaluation of 53 nodes in 76 steps instead of 1529 nodes published in [5].

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